Inertial mass of an elementary particle from the holographic scenario

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Abstract

Various attempts have been made to fully explain the mechanism by which a body has inertial mass. Recently it has been proposed that this mechanism is as follows: when an object accelerates in one direction a dynamical Rindler event horizon forms in the opposite direction, suppressing Unruh radiation on that side by a Rindler-scale Casimir effect whereas the radiation in the other side is only slightly reduce by a Hubble-scale Casimir effect. This produces a net Unruh radiation pressure force that always opposes the acceleration, just like inertia, although the masses predicted are twice those expected, see [17]. In a later work an error was corrected so that its prediction improves to within 26% of the Planck mass, see [10]. In this paper the expression of the inertial mass of a elementary particle is derived from the holographic scenario giving the exact value of the mass of a Planck particle when it is applied to a Planck particle.

Keywords: inertial mass; Unruh radiation; holographic scenario, Dark matter, Dark energy, cosmology.
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PACS 04.62.+v - Quantum fields in curved spacetime
PACS 06.30.Dr - Mass and density

1 Introduction

The equivalence principle introduced by Einstein in 1907 assumes the complete local physical equivalence of a gravitational field and a corresponding non-inertial (accelerated) frame of reference (Einstein was thinking of his famous elevator experiment). In a similar way we can assume a holographic equivalence principle where it is the same to have a particle accelerated because it is attracted by a central mass than a particle accelerated by an event horizon. The question of why a particle is accelerated towards an event horizon has two different answers. In the Verlinde's holographic model, see [26], the acceleration of the particle towards the event horizon is due to the entropic force arising from thermodynamics on a holographic screen (the event horizon). The entropic force appears in order to increase the general entropy according to the second law of thermodynamics. However we can also think that the radiation from the region of space behind this event horizon can never hope to catch the particle causing a real imbalance in the momentum transferred by all the radiation from all directions which produces an acceleration of the particle towards the event horizon, see [10, 17]. Both arguments are claims because are based in the existence of effects not universally accepted. If in a future they are proved then we will accept that there will be a complete physical equivalence between a gravitational field and a corresponding event horizon. This holographic equivalence principle would be the base of a new gravitational theory where gravity will be emerging from an holographic scenario from a dynamical point of view. In this work, we can establish the origin of the inertial mass of a elementary particle from the holographic scenario. The problem of the inertial mass of a macroscopic body is still open in this context. First we recall some concepts.

The Hawking radiation, predicted by Hawking [14] in 1974, is black-body radiation to be released by black holes due to quantum effects near the event horizon of the black hole. The vacuum fluctuations cause a particle-antiparticle pair to appear close to the event horizon. One of the pair falls into the black hole while the other escapes. The particle that fell into the black hole must had negative energy in order to preserve total energy. The black hole loses mass because for an outside observer the black hole just emitted a particle.

The Unruh effect [25] is the prediction that an accelerating observer will observe black-body radiation where an inertial observer would observe none. A priori the Unruh effect and the Hawking radiation seem unrelated, but in both cases the radiation is due to the existence of an event horizon. In the case of the Unruh radiation, on the side that the observer is accelerating away from there appears an apparent dynamical Rindler event horizon, see [19]. The appearance of this event horizon produces two effects: a radiation in a similar way to the Hawking radiation from the horizon and a force toward the horizon that accounts for the inertial mass of the elementary particle (see below). Therefore an accelerating observer perceives a warm background whereas a non-accelerated observer will see a cold background with no radiation.

Various attempts have been made to fully explain the mechanism by which a body has inertial mass, see for instance [3] where the principle of equivalence is examined in the quantum context. We recall that the relativistic mass [24] is the measure of mass dependent on the velocity of the observer in the context of the special relativity but is not an explanation of the rest mass. In [17] an origin of the inertia mass of a body was suggested: for an accelerated particle the Unruh radiation becomes non-uniform because the Rindler event horizon reduces the energy density in the direction opposite to the acceleration vector due to a Rindler-scale Casimir effect whereas the radiation on the other side is only slightly reduced by a Hubble-scale Casimir effect due to the cosmic horizon. Therefore there is an imbalance in the momentum transferred by the Unruh radiation and this produces a force which is always opposed to the acceleration, like inertia. In [10] it is corrected a mistake detected in [17]. The correct expression for the force is

$$F_x = -\frac{\pi^2 h a}{48 c l_p}, \tag{1}$$

where $l_p = 1.616 \times 10^{-35} m$ is the Planck distance. Hence the inertial mass is given by $m_i \sim \pi^2 h/(48 c l_p) \sim 2.75 \times 10^{-8} kg$ which is 26% greater than the Planck mass $m_p = 2.176 \times 10^{-8} kg$.

In this paper we derive an expression for the inertia of an elementary particle from the holographic scenario, giving the exact value of the mass of the Planck particle when it is applied to this Planck particle.

2 Holographic scenario for the inertia

The holographic principle proposed by 't Hooft states that the description of a volume space is encoded on a boundary to the region, preferably a light-like boundary like a gravitational horizon, see [23]. This principle suggests that the entire universe can be seen as a two-dimensional information structure encoded on the cosmological horizon, such that the three dimensions we observe are only an effective description at macroscopic scales and at low energies. Verlinde proposed a model where the Newton's second law and Newton's law of gravitation arise from basic thermodynamic mechanisms. In the context of Verline's holographic model, the response of a body to the force may be understood in terms of the first law of thermodynamics. Indeed Verlinde conjecture that Newton and Einstein's gravity originate from an entropic force arising from the thermodynamics on a holographic screen, see [26]. Moreover the holographic screen in Verlinde's formalism can be identified as local Rindler horizons and it is suggested that quantum mechanics is not fundamental but emerges from classical information theory applied to these causal horizons, see [15, 16].

An important cosmological consequence is that at the horizon of the universe there is a horizon temperature given by

$$T_H = \frac{\hbar H}{2\pi k_B} \sim 3 \times 10^{-30} K, \tag{2}$$

and this temperature has associated the acceleration a_H given by the Unruh [25] relationship

$$a_H = \frac{2\pi c\, k_B T_H}{\hbar}, \tag{3}$$

and substituting the value of T_H we arrive to $a_H = cH \sim 10^{-9} m/s^2$ in agreement with the observation. The entropic force pulls outward towards the horizon apparently creating a Dark energy component and the accelerated expansion of the universe, see [4, 5].

Due to the existence of the cosmic horizon all the matter of the universe is attracted by the horizon comparable to the Hubble horizon due to the entropic force and accelerated towards this horizon with an acceleration given by Eq. 3. However this acceleration is ridiculously small compared to local acceleration due to nearby bodies and it is only relevant for isolated bodies with very low local accelerations for instance a star at the edge of a galaxy giving also an explanation to the obtained rotation curves. First you fix an observer and equation (3) gives the acceleration that any body feels toward the horizon in the direction far away from the observer. Moreover this acceleration is ridiculously small compared with the local acceleration of bodies at small distance where the local movement is the relevant. For instance the movement in collision of our galaxy with the Andromeda galaxy. However for distant bodies, where the local movement is irrelevant for an observer so far, the accelerate expansion is relevant and we see that these bodies accelerate outside from the observer. Additionally in an accelerating universe, the universe was expanding more slowly in the past than it is today.

Therefore the total acceleration measured by an observer is $a = a_L + a_H$ where a_L is the local acceleration due to the local dynamics that suffers a particle. It is clear that only for very low local movements the acceleration a_H becomes important. We can assume that the local movement is the gravitational attraction of a central mass and then we have

$$a - a_H = a_L = \frac{GM_\odot}{r^2}. \tag{4}$$

Equation (4) can be written into the form

$$a\left(1 - \frac{a_H}{a}\right) = \frac{GM_\odot}{r^2}. \tag{5}$$

Hence, following [9] (see also [7]) for low local accelerations we obtain a modified inertia given by

$$m_I = m_i\left(1 - \frac{a_H}{a}\right) = m_i\left(1 - \frac{2\pi c\, k_B T_H}{\hbar a}\right), \tag{6}$$

where m_i is the inertial mass and m_I is the modified inertial mass. This modified inertial mass has the MOND requirements to explain the galaxies rotation curves problem and obviating the Dark matter. The requirements of MOND arise from the fact that while Newton's laws have been extensively tested in high-acceleration environments (in the solar system and on earth), they have not been verified for objects with extremely low acceleration, such as stars in the outer parts of galaxies. In [18] Milgrom propose a new effective gravitational force law that reduces to the second Newton's law at high acceleration but lead to different behavior at low acceleration. More precisely, it has the form

$$m_i\, \mu(a/a_0)\, \mathbf{a} = \mathbf{F} \tag{7}$$

where $\mu(x \gg 1) \approx 1$, and $\mu(x \ll 1) \approx x$, $a = |\mathbf{a}|$ and a_0 is the acceleration constant, replacing the classical form $m_i\ \mathbf{a} = \mathbf{F}$. For accelerations much larger than the acceleration constant a_0, we have $\mu \approx 1$, and Newtonian dynamics is restored. However for small accelerations $a \ll a_0$ we have that $\mu = a/a_0$. In fact in equation (5) the constant acceleration a_0 is a_H and in the case $a \gg a_H$ we obtain the second Newton law and for $a \ll a_H$ we are in the deep-MOND regime. In this case the star's rotation velocity is independent of its distance from the center of the galaxy, the rotation curve is flat, as it is required.

In equation (6) obviously the T_H can be interpreted as the temperature of the horizon comparable to the Hubble horizon that in the context of the Verlinde's theory produces an entropic force. Equation (6) is the same equation that obtained McCulloch, see for instance [10, 17],

$$m_I \sim m_i \left(1 - \frac{2c^2}{a\Theta}\right), \tag{8}$$

because $2c^2/\Theta = c^2/R_U$ because $\Theta = 2R_U$. And taking into account that $R_U = c/H$ we have that $2c^2/\Theta = cH = a_H$. Other approach to explain the Dark matter and the Dark energy is assuming that the antiparticles have negative gravitational charge and consequently the quantum vacuum, well established in the standard model of particles and fields, contains virtual gravitational dipoles that produce the phenomena desired, see [11]. In addition to theory there are now emerging astronomical measurements to test the eventual gravitational effects of quantum vacuum, see [6, 12].

Our model is based in the Unruh radiation resulting from the acceleration of the elementary particle with respect to surrounding matter. We will see how to derive the value of the inertial mass of an elementary particle from the holographic scenario. The deduction of the inertial mass is directly the application of entropy principles and thermodynamics to the particle event horizon. The particle event horizon is a consequence of treat elementary particles as a black hole in the context of the field interaction of the particle. We recall that any elementary particle is, in fact, a singularity of the field. This generalization to any field interaction is not new. The so-called *strong gravity* is an example. The scale invariance of general relativity is applied to the strong gravity [20, 21, 1, 22] that tries to derive the hadron properties from a scaling down the gravitational theory, treating particles as black-hole type solutions of the strong field. The hadron might be considered as peculiar strong black hole using the Einstein-type equations

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}^\rho_\rho - H\tilde{g}_{\mu\nu} = -\frac{8\pi G_s}{c^4}S_{\mu\nu}, \qquad 2H \equiv \left(\frac{m_s c}{\hbar}\right)^2, \tag{9}$$

where G_s is the universal constant associated to the strong field and H is the hadronic constant with m_s the mass of the external strong quanta. Equation (9) is obtained from the classical Einstein equations by a convenient covariant dilatation, see [1]. In the gravitational case, for a stationary spherically symmetric mass distribution M, we get in vacuum the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} + \frac{\Lambda r^2}{3}\right)dt^2 - \left(1 - \frac{2GM}{c^2 r} + \frac{\Lambda r^2}{3}\right)^{-1} dr^2 - r^2 d\Omega. \tag{10}$$

If we write equation (11) in an explicitly dilatation-covariant way we obtain

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\varrho + \frac{\Lambda r^2}{3\varrho^2}\right) dt^2 - \left(1 - \frac{2GM}{c^2 r}\varrho + \frac{\Lambda r^2}{3\varrho^2}\right)^{-1} dr^2 - r^2 d\Omega. \quad (11)$$

where $\varrho = 1$ in the gravitational case and $\varrho \approx 10^{-40}$ in the strong case. In this last case $G_s = G/\varrho$ and $\Lambda/\varrho^2 \equiv H$. To get the Schwarzschild radii, we need essentially to solve the equation

$$1 - \frac{2GM}{c^2 r} + \frac{\Lambda r^2}{3} = 0 \quad (12)$$

which always admits only one solution, that in the case of the cosmos as gravitational black hole is

$$r_s^{(G)} \simeq 2GM/c^2 \approx 10^{26} m. \quad (13)$$

In dilatation-covariant form equation (12) reads as

$$\eta^3 + \frac{3}{\Lambda}\eta - \frac{6GM}{c^2 \Lambda} = 0 \quad (14)$$

where as usual $\eta \equiv r/\varrho$. In the strong case, with $\varrho \approx 10^{-40}$ one obtain the hadrons radii is given by $r_s^{(S)} = 10^{-40} \cdot 10^{26} = 10^{-14} m.$

However we will see that this framework will be applied at Planck scale which is where the framework of the strong gravity has sense. Moreover the result will be applied, as we will see, at a Planck particle.

Hence, in this framework we treat any particle of mass M_i as a black-hole. This black-hole particle has an event horizon: the *particle event horizon,* at the Schwarzschild radius. The entropy on this particle event horizon is given by

$$S = \frac{k_B c^3 A}{4G\hbar} = \frac{k_B c^3 \pi r_s^2}{G\hbar}, \quad (15)$$

Because the area $A = 4\pi r_s^2$, where r_s is the radius of the particle event horizon of the particle of mass M_i. The incremental ratio respect to r is

$$\frac{dS}{dr} = \frac{k_B c^3 2\pi r}{G\hbar}, \quad (16)$$

The entropic force is given by

$$F = \frac{dE}{dr} = T\frac{dS}{dr}. \quad (17)$$

The associated temperature outside of the particle event horizon that feels any other particle of mass m_i due to the Hawking radiation is

$$T = \frac{1}{2\pi}\frac{\hbar g}{k_B c}, \quad (18)$$

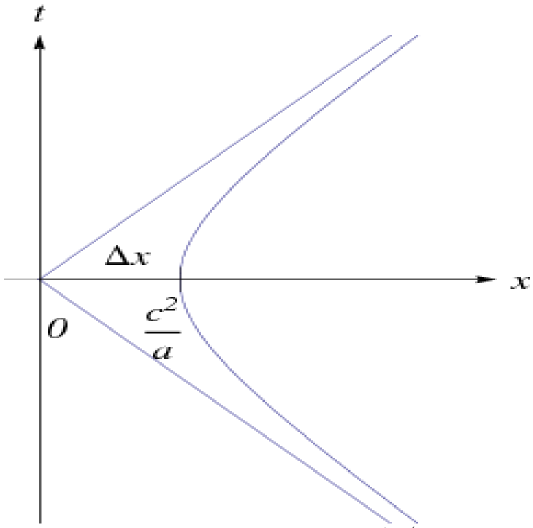

Figure 1: The Rindler horizon to its left (at a distance c^2/a away).

where g is the gravitational acceleration of this particle of mass m_i towards the original one M_i. The inertial mass of the particle M_i is manifested if this particle M_i is accelerated. However, the presence of the particle m_i also accelerates the particle of mass M_i and we have $M_i a = m_i g$ according to the fact that action and reaction are equal. The acceleration of the particle M_i produces a Rindler event horizon to its left whose associated temperature due to the Unruh effect is given by

$$T = \frac{1}{2\pi}\frac{\hbar a}{k_B c}. \tag{19}$$

Now we compute the entropy on this Rindler horizon which is given as before by the equation (15). But now the area is not the area of a sphere. In Figure 1 we can see the Rindler horizont is given by the surface $x = t$ and for $t = 0$ the distance the Rindler horizon to the particle is $\Delta x = c^2/a$. In Figure 1 the spatial coordinates are reduced to a single axes x. But, in fact, the Rindler horizon is given by all the the spatially points at distance c^2/a at the left of the particle. This gives approximately a half sphere. Therefore now the horizon area is the half that in the case of a black hole, that is, $A = 4\pi r^2/2$. Therefore the entropy results

$$S_R = \frac{k_B c^3 A}{4G\hbar} = \frac{k_B c^3 \pi r^2}{2G\hbar}. \tag{20}$$

In this case the incremental ratio respect to r is

$$\frac{dS_R}{dr} = \frac{k_B c^3 \pi r}{G\hbar}. \tag{21}$$

Substituting into equation (17) and taking into account (19) and (21) we have

$$F = M_i a = \frac{c^2 r a}{2G}. \tag{22}$$

Consequently the inertial mass M_i of the elementary particle is given by

$$M_i = \frac{c^2 r}{2G}. \tag{23}$$

This result is not surprising because isolating r we obtain $r = 2GM_i/c^2$ which is the Schwarzschild radius of the elementary particle, just a consequence of having treated the particle as a black hole. However is the first time that the inertial mass of an elementary particle is expressed in function of the Schwarzschild radius using the holographic scenario.

The de Broglie wavelength for the massive particle is $\lambda = \hbar/(mc)$, while the Schwarzschild radius for such a black hole is $r_s = 2Gm/c^2$. Thus these two lengths become equal when m is the mass of a Planck particle. When this happens, they both equal the length of a Planck particle $l_p = \sqrt{2Gh/c^3}$. So to make sense out of this identification one has to imply that $r_s \sim \lambda$ which is only possible for $m \sim m_p$. It is well known on the other hand that black hole with masses less than m_p would have a temperature larger than T_p signaling a breakdown of the semiclassical Hawking calculation. Hence we would like to stress that for any particle of mass less than m_p it is not possible to apply our development. If we substitute in (23) the radius of the particle r by the radius of a Planck particle $r_p = \sqrt{2Gh/c^3}$ we obtain that the expression of the inertial mass (23) becomes

$$M_i = \frac{c^2 r_p}{2G} = \frac{c^2\sqrt{2Gh/c^3}}{2G} = \sqrt{\frac{hc}{2G}}, \tag{24}$$

which coincides with the value of the mass of a Planck particle m_p. This mass is $\sqrt{\pi}$ times larger than the Planck mass, making a Planck particle 1.772 times more massive than the Planck unit mass.

For the case that we consider all the universe with the cosmic horizon or comparable Hubble horizon we have that $M_i \equiv M_U$ and $r \equiv R_U$. We can also consider all the universe as a black hole. The entropy on the comparable Hubble horizon is

$$S_H = \frac{k_B c^3 A}{4G\hbar} = \frac{k_B c^3 \pi R_H^2}{G\hbar}. \tag{25}$$

The incremental ration respect to the radial variable is

$$\frac{dS}{dr} = \frac{k_B c^3 2\pi R_H}{G\hbar}. \tag{26}$$

Substituting into equation (17) the acceleration associated to the horizon temperature (2) and equation (22) we obtain

$$M_U = \frac{c^2 R_U}{G}. \tag{27}$$

In this case the movement of the masses of the universe towards the horizon modifies the entropy of the comparable Hubble horizon. Condition (27) that links the mass of the universe and its radius

$$GM_U = c^2 R_U, \tag{28}$$

was obtained before in different contexts, see for instance [8] and references therein. Moreover using (17) and (2) and taking into account that $R_U = c/H$ we have that

$$F = -\frac{dE}{dr} = -T_H \frac{dS}{dr} = -\frac{c^4}{G}. \tag{29}$$

This entropic force towards the Hubble horizon explains the acceleration of universe expansion without the need of a negative pressure of a dark energy. The pressure exerted by this entropic force is

$$P = \frac{F}{A} = -\frac{c^4}{4\pi G R_U^2} = -\frac{c^2 H^2}{4\pi G} = -\frac{2}{3}\rho_c c^2.$$

where $\rho_c = 3H^2/(8\pi G)$ is the critical energy density and this value is the currently measured dark energy/cosmological constant value, see [4] where this last development was obtained by first time. A comparative of two models of entropic force with their associated entropic acceleration and several ΛCDM models is made in [4] (see in particular Figure 1 in [4]).

3 Conclusion

In [17] was shown that inertial mass of a body can be derived by assuming that inertia is caused by the formation of a Rindler horizon behind an object as it accelerates, which suppresses the radiation on that side of the object and pushes it back again the acceleration. In [10] an error made in [17] was corrected but still does not give the correct value of the Planck mass. This suggest that the development used in [10, 17] not give correct results for an elementary particle. In [10] it is also suggested that inertia can be understood as an attempt by the system to equalize Unruh temperatures between the background cosmic value and that due to local accelerations.

In this work an expression for inertia of an elementary particle is derived from the holographic scenario giving the correct value of the mass of a Planck particle when is applied to this particle. The case of a macroscopic body cannot be treated in the context of the theory developed in this work because a macrocosmic body is not a black hole. For instance the Sun has radius approximately $700.000 Km$ and a black hole with its mass would have a radius (the event horizon radius) about $3Km$. Therefore the formula that give the entropy (15) that express the entropy of a black hole cannot be applied to a macroscopic body as the Sun. The development for a macroscopic body must take into account that the constituents of a body that are elementary particles and all these particles can be treated as black holes. Hawking [13] showed under general conditions that the total area of the event horizons of any collection of classical black holes can never decrease, even if they collide and merge. Using these ideas could be possible to obtain the inertial mass of a macroscopic body in the context of the holographic scenario.

We also remark that the discussion of the inertial mass is applied for a Planck particle and not for any elementary particle. The assumption that elementary particles are black holes is an old assumption [20, 21, 1, 22] that has several problems not solved up to now. For instance, for a particle like an electron attending to its mass the event horizon would be at about $10^{-57}m$, in the classical Scwharzschild geometry. Moreover in the Reissner-Nordström metric, which takes into account the charge of the electron, it would be larger at about $10^{-37}m$. Consequently these two values are inconsistent and implies that if the electron were a black hole, it would be a naked singularity, which is thought to be inconsistent theoretically.

Moreover Quantum Electrodynamics that agrees with high precision with experiments, treats the electron as if it had exactly zero radius, and suggest that the electron has no substructure on scales at least small than $10^{-22}m$. However no one knows what happens on scales of $10^{-37}m$ because no experiments can be done at such distance scales. It is clear that if any elementary particle is black hole, it would have to have a mass near to the Planck mass (in fact is its definition) and be described by quantum gravity. It is for this reason that the work is focused to a Planck mass. Moreover in this tiniest scale, the Planck scale, gravity regains its principal key role. For instance in the string theory gravity plays a stronger role in higher dimensional space and it is only in our four dimensional space that gravity appears so weak. This extra dimensions become important only on the Planck scale and at this level is when the black holes as elementary particles become a possibility. Recently Coyne and Cheng [2] have studied the properties of the black holes on that scale and they predict the existence of huge numbers of black hole particles at different energy levels. In fact the authors propose a model of evaporation of black holes that, under certain circumstances, certain black holes would be indistinguishable from the elementary particles. According to the authors, the complete evaporation of a black hole would end up leaving a remnant described by quantum mechanics. It may be thought that a black hole is gradually transformed into an unstable particle that decays into other particles until it becomes an elementary particle.

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